Even dimensional submanifolds of spheres with nonnegative curvature operator

Nondas E. Kechagias

Abstract

We consider even dimensional submanifolds of spheres with non negative curvature operator satisfying a certain restriction on their Ricci curvature defined by T. Vlachos. They are homeomorphic to a sphere, a product of two spheres, or the complex projective space of dimension 2.

1 Introduction

Relations between curvature and topology of Riemannian manifolds have been under investigation for many years. After the beautiful theorem of Myers relating the Ricci curvature of a complete n-dimensional Riemannian manifold M with compactness and finiteness of the fundamental group, a number of versions of the sphere theorem have appeared in the literature.

Much of work has been done recently concerning the topology of a submanifold M of the unit sphere with positive Ricci curvature.

Extending an idea of Synge, Lawson and Simons related the topology of a compact Riemannian manifold M^n isometrically immersed into a space form $F^{n+p}(c)$ of constant non-negative sectional curvature with stable currents ([9]). Around the same time Gallot and Meyer ([5]) extending a well known result of De Rham which permits the decomposition of a complete, connected, simply connected Riemannian manifold with non-negative curvature operator into product of irreducible factors also related the topology with curvature. Following Lawson and Simons, Leung ([10]) considered minimal submanifolds of codimension l in the unit sphere S^{n+l}. Using a similar method as of Leung, Shiohama and Xu ([11]) extended his result "improving" his bound. Under the same pattern Hasanis and Vlachos ([7]) proved the analogue theorem of Leung for odd dimensional submanifolds using a bound of the Ricci curvature.

Theorem 1 (Hasanis and Vlachos) *Let M be an odd n-dimensional compact minimal submanifold of the unit sphere S^{n+l}. Assume that the Ricci curvature satisfies $Ric > \frac{n(n-3)}{n-1}$. Then i) if $n > 3$, M is homeomorphic to a sphere; ii) if $n = 3$, then M is topologically a space form of positive sectional curvature.*

be non-zero in at least three different degrees. Thus $M = M_1 \times M_2$. Because of the restrictions on M_t, the fact that $H^i(M,\mathbb{Z}) = H^{2m-i}(M,\mathbb{Z}) = 0$ for $i \neq m$ and the Universal Coefficient theorem, we conclude that $H^i(M_t,\mathbb{Z}) = H^i(S^m,\mathbb{Z})$ for all i. It follows that $M_t \equiv S^m$. Our theorem follows.

Remark 10 *The case $l = 1$ has been studied by Baldin and Mercuri in [1] without restriction on the Ricci curvature.*

References

[1] Y. Baldin and F. Mercuri, Isometric immersions in codimension two with non-negative curvature, Math. Z., 173, 1980, 111-117.

[2] D. Barden, Simply connected five manifolds. Annals of Mathematics, 82, 1965, 365-385.

[3] B. Chow and D. Yang, Rigidity of Nonnegatively Curved Compact Quaternionic-Kähler Manifolds. J. Differential Geometry, 29, 1989, 361-372.

[4] M. H. Freedman and Quinn, **Topology of 4-Manifolds.** Princeton University Press, 1990.

[5] S. Gallot and D. Meyer, Opérateur de courbure et Laplacien des Formes differentielles d'une variété riemannienne. J. Math. Pures Appl. 54, 1975, 259-284.

[6] R. S. Hamilton, Four-manifolds with positive curvature operator. J. Differential Geometry, 24, 1986, 153-179.

[7] T. Hasanis and T. Vlachos. Ricci curvature and minimal submanifolds, Pacific Journal of Mathematics, Vol. 197, No. 1, 2001, 13-24.

[8] Z. Hu and S. Zhai. A sphere theorem for even-dimensional submanifolds of the unit sphere. J. Zhenqzhou Univ. Mat. Sci. Ed., 37, 2005, no. 1, 1-4.

[9] B. Lawson and J. Simons, On stable currents and their application to global problems in real and complex geometry. Annals of Mathematics, 98, 1973, 427-450.

[10] P-F Leung, Minimal submanifolds in a sphere. Math. Z. 183, 1983, 75-86.

[11] K. Shiohama and H. Xu, The topological sphere theorem for complete submanifolds. Compositio Mathematica, 107, 1997, 221-232.

[12] D. Sjerve, Homology spheres which are covered by spheres. J. London Math. Soc. (2), 6, 1973, 333-336.

[13] T. Vlachos, A sphere theorem for odd-dimensional submanifolds of spheres. Proc. Amer. Math. Soc. 130 (2002), no. 1, 167–173.